2017
PHYSICS – HONOURS
First Paper
Full Marks – 100

The figures in the margin indicate full marks
Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No. 1*** and ***any four*** questions from each **Unit**

1. Answer ***any ten*** of the following : 2× 10

(a) Find the values of x for which the series $\sum_{n=1}^{\infty}\frac{x^n}{n+1}$ is convergent.

(b) Prove that $\int_V\left(\phi\nabla^2\psi+\nabla\phi.\nabla\psi\right)dV=\oint_S(\phi\nabla\psi)d\vec{S}$ where V is the volume enclosed by the closed surface S.

(c) Prove that the product of two Hermitian matrices A and B is Hermitian only if A and B commute.

(d) Using the generating function

$\phi(x,h)=\left(1-2xh+h^2\right)^{-1/2}=\sum_{l=0}^{\infty}h^lP_l(x)$, $(|h|<1)$ find out the Legendre Polynomial $P_2(x)$.

(e) Find the Fourier transform of $\delta(x-a)+\delta(x+a)$ where a is a constant.

(f) Find whether $d\phi$ is an exact differential where

$d\phi=\left(x^3-4xy\right)dx+\left(y^2-2x\right)dy$.

(g) Distance between two points in a medium is $2m$. The optical path corresponding to this distance is $2.5m$. Find out the velocity of light in the medium.

(h) Distinguish between amplitude resonance and velocity resonance.

(i) Draw a sketch of the drain characteristics of a MOSFET and identify the different regions.

(j) Verify the Boolean identity

$AC+ABC=AC$

(k) Why FET's can be used at higher frequencies than BJT's?

(l) A dice is thrown 8 times. What is the probability that 4 will show (i) exactly twice (ii) at least 6 times?

Unit – I

2. (a) (i) Deduce $\vec{\nabla}\times\left(\phi\vec{A}\right)=\left(\vec{\nabla}\phi\right)\times\vec{A}+\phi\left(\vec{\nabla}\times\vec{A}\right)$.

(ii) State Stokes' theorem for a vector $\vec{V}$. Let $\vec{V}=\phi\vec{c}$ where $\vec{c}$ is a constant vector. Use the result of (i) to deduce that $\int\vec{\nabla}\phi\times d\vec{s}=-\oint\phi d\vec{r}$. 2+1+3

(b) Given the vector $\vec{A}=\left(x^2-y\right)\hat{i}+2x\hat{j}+2\hat{k}$, evaluate $\oint\vec{A}.d\vec{r}$ around the boundary of the circle $x^2+y^2=1$. 4

[Turn Over]

3. (a) Show that any square matrix can be written as the sum of a symmetric and antisymmetric matrix. 2

(b) Find the eigenvalues and the normalised eigenvectors of the matrix $M = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$. 2+3

(c) A normal matrix N is defined by the relation $NN^{+} = N^{+}N$. If N is written as $(A + iB)$, where A and B are Hermitian, show that A and B commute. 3

4. (a) For free path of length x during which a molecule of an ideal gas does not suffer any collision with another molecule, the probability distribution function is given by $P = \frac{1}{\lambda} e^{-x/\lambda}$, $0 < x < \infty$. Show that the mean free path is λ. 3

(b) Expand $\ln(1+x)$ in a Taylor series about the point $x = 0$. Hence write down the nth term of the series. 3

(c) Evaluate $\iint xy\,dxdy$ over the area between $y = x^2$ and $y = x$. 4

5. (a) Apply Frobenius method to solve the equation

$$\frac{d^2y}{dx^2} + y = 0, \text{ setting } y(x) = \sum_{\lambda=0}^{\infty} a_{\lambda} x^{k+\lambda} \text{ with } a_0 \neq 0.$$

(i) Verify that the indicial equation is $k(k-1) = 0$.

(ii) For $k = 1$, show that a_1 is necessarily zero.

(iii) Find the recurrence relation.

(iv) Hence show that for $k = 1$, the solution is $y = a_0 \sin x$. 2+1+2+2

(b) A particle moves in a plane such that

$$\frac{dx}{dt} = -50y \text{ and } \frac{dy}{dt} = 18x.$$

Given that it passes through the point $(2,0)$, find the equation of the path. 3

6. (a) Solve $\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y}$ by the method of separation of variables, given that $U(0,y) = 8e^{-3y}$. 4

(b) Given

$$e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}$$

where H_n is the Hermite Polynomial, show that $H_n(x) = (-1)^n H_n(-x)$. 2

(c) A plucked string of length L is excited at $x = L/3$ and touched at $x = L/4$. Calculate the harmonics present. 4

7. (a) Expand the function

$$f(x) = x^2 \text{ for } -\pi < x < \pi$$
$$f(x + 2\pi) = f(x)$$

in a Fourier series. Hence prove, $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$. 4+2

(b) Show that the Fourier transform of $f(x)=e^{-|x|}$ is $F(k)=\sqrt{\frac{2}{\pi}}\frac{1}{1+k^2}$. 4

Unit – II

8. (a) Deduce the Lagrange-Helmholtz relation connecting lateral and angular magnification of an optical system. 4

(b) Using the paraxial approximation, construct the system matrix for a thin lens made with a material of *RI* μ and radii of curvature R_1 and R_2 respectively. 4

(c) Show that the planar surface of a plano-convex lens does not contribute to the system matrix. 2

9. (a) A mass of 1 kg is acted on by a restoring force with force constant 4N/m and a resisting force with damping coefficient 2N-s/m. Write down the equation of motion in one dimension. Find :

(i) whether the motion is periodic or oscillatory.

(ii) the value of the resisting force which will make the motion critically damped. 1+2+2

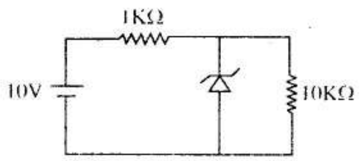
(b) Define group velocity and phase velocity. 2

(c) The dispersion relation for transverse waves propagating in a medium is given by $\omega^2=\omega_p^2+k^2c^2$ where the symbols have their usual meanings. Show that $v_g v_p = c^2$. 3

10. (a) A particle is subjected to two SHMs at right angles to each other having the same frequency. Show that the resultant locus of the particle is an ellipse. Hence find the locus when the two motions are in phase and in the opposite phase. 3+1+1

(b) A number of SHMs, all in the same straight line and having equal amplitude and frequency but an equal phase difference θ between the consecutive SHMs are superimposed. Calculate the amplitude and phase of the resultant motion using complex form of representation. 5

11. (a) In the circuit below, what are the currents that flow through $1\text{K}\Omega$, $10\text{K}\Omega$ resistors and the Zener diode? Assume that the Zener has a breakdown voltage 6V, what will be the value of current if the $10\text{K}\Omega$ resistor is replaced by $500\,\Omega$ resistor? What is the voltage across the Zener now? Consider the Zener to be an ideal device. 2+2+1



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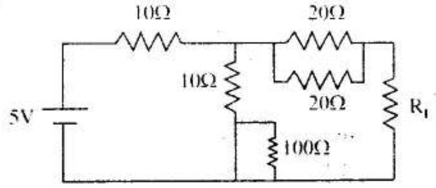
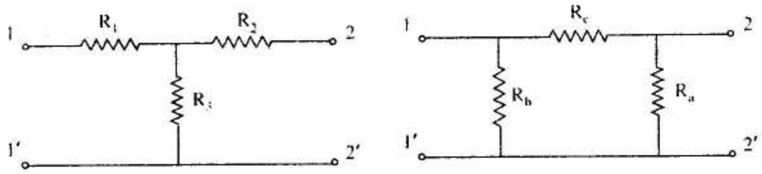
(b) Calculate the voltage drop and power dissipated across R_L in the circuit below, using Thevenin's theorem. Find the value of R_L for which power dissipated across R_L is maximum. 4+1



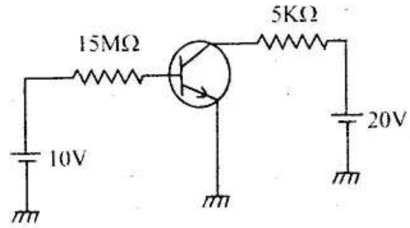
12. (a) Consider the T and π networks.



Show that the networks will be equivalent when

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c},\ R_2 = \frac{R_c R_a}{R_a + R_b + R_c},\ R_3 = \frac{R_a R_b}{R_a + R_b + R_c}.$$ 5

(b) For a transistor in CE configuration given below, it is found that for a fixed base current of 30 μA, I_C changes from 3.5mA to 3.7mA when V_{CE} changes from 7.5V to 12.5V. Calculate its output resistance and β at $V_{CE} = 12.5$V. What will be the value of α? 5



13. (a) A function of 3 Boolean variables have high output either when all 3 inputs are high or any one input is high. Write down the Truth Table and derive the simplified Boolean expression. 2+2

(b) Subtract $(110101)_2$ from $(1110110)_2$ using 2's complement method. 2

(c) Obtain the Boolean expression for the output 'y' of the logic circuit given below. 4